Modeling Global Structural Damping in Trusses Using Simple Continuum Models

C. T. Sun*
Purdue University, West Lafayette, Indiana
and
J. N. Juang†
NASA Langley Research Center, Hampton, Virginia

Truss beams with members having viscous damping are modeled as continuum Timoshenko beams. Procedures for deriving the equivalent beam stiffnesses and damping are presented. The global damping for the continuum beam is explicitly expressed in terms of the damping coefficients of the individual truss members. The continuum beam model is used to study transient vibration problems and the solutions compare well with the full-scale finite element solutions. The gradient method is used for parameter estimations in conjunction with the Timoshenko beam model. It is shown that the Timoshenko beam model can be updated easily with measured data and that the updated model can yield very accurate transient solutions.

I. Introduction

IN the special space environment where air damping is absent, a large space structure can derive damping only from structural deformations. There are two main sources of structural damping, i.e., the material damping of the structural members and the damping originating from friction at the joints. If the joints are designed rigid, then the material damping may provide the sole source of structural damping in the system.

Because of their enormous sizes, large space structures are often replaced by equivalent simple continuum models for dynamic analyses.[1-4] Such simple continuum models have been shown to accurately predict the natural frequencies and mode shapes in elastic structures.[1] They also have been found convenient to use in problems involving identification of structural parameters in large structures.[3] The purpose of this paper is to derive the structural damping for the simple continuum model based on the damping properties of the individual structural members.

Detailed procedures for deriving the equivalent elastic beam rigidities from a truss beam were given by Ref. 1. Among the three beam models (shear, Bernoulli-Euler, and Timoshenko), the Timoshenko beam theory was found to be most accurate and most suitable to represent general trusses. In this study, the Timoshenko beam model is used.

During its service life, the structural properties of a space structure may need verification from time to time. Such updating is necessary for the control systems to work. In this paper, the Timoshenko beam is used for identification of structural parameters based upon "on-site" measurement data.[5,6] The gradient method is employed to perform the parameter estimation needed when updating the Timoshenko beam model.

II. Timoshenko Beam Model

As an illustration for the procedure, consider a truss beam symmetric with respect to the midplane so that the extension is not coupled with the flexural deformation. In the absence of damping, the shear force Q and the bending moment M are related to the transverse displacement w and the rotation ψ as

$$\begin{Bmatrix} Q \\ M \end{Bmatrix} = \begin{bmatrix} \overline{GA} & 0 \\ 0 & \overline{EI} \end{bmatrix} \begin{Bmatrix} w,_x + \psi \\ \psi,_x \end{Bmatrix} \tag{1}$$

where $\overline{GA}$ is the transverse rigidity and $\overline{EI}$ the bending rigidity. In Eq. (1), a comma indicates partial differentiation.

$\overline{EI}$ and $\overline{GA}$ are obtained from a typical substructure by applying a bending moment $1 \times L_g$ and a unit shear force, respectively, as shown in Fig. 1. They are expressed as

$$\overline{EI} = L_g L_c / \phi, \qquad \overline{GA} = 1/\theta \tag{2}$$

in which the deformations ϕ and θ are produced by the applied moment and the applied shear force, respectively. The values of ϕ and θ can be calculated analytically or numerically by using the matrix method. These analyses also yield uniaxial strains in the truss members. Using the substructure in Fig. 1 as an example, the member strains corresponding to the bending deformation are given formally as

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_6 \end{Bmatrix} = \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_6 \end{Bmatrix} \psi,_x \tag{3}$$

and, corresponding to the transverse shear deformation, we have

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_6 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_6 \end{Bmatrix} (w,_x + \psi) \tag{4}$$

where the subscripts indicate the truss members as shown in Fig. 1. In Eqs. (3) and (4), the coefficients a_i and b_i depend on the member geometry and material properties, and the rotation gradient $\psi,_x$ and the transverse shear strain $w,_x + \psi$

Received Oct. 31, 1983; revision received March 29, 1985.

*Professor, School of Aeronautics and Astronautics.

†Aerospace Technologist.

are approximated by

$$\psi,_x = \phi / L_c \tag{5}$$

and

$$w,_x + \psi = \theta \tag{6}$$

respectively.

If the truss members are symmetric with respect to the midplane, i.e.,

$$A_1 = A_2, \quad A_3 = A_4, \quad A_5 = A_6$$

$$E_1 = E_2, \quad E_3 = E_4, \quad E_5 = E_6 \tag{7}$$

where A_i and E_i are the cross-sectional area and the Young's modulus of member i, respectively, then the coefficients a_i and b_i in Eqs. (3) and (4) can be obtained as

$$a_1 = L_g/2, \; a_2 = -a_1, \; a_3 = a_4 = a_5 = a_6 = 0 \tag{8}$$

and

$$b_3 = L_g L_c / (L_g^2 + L_c^2), \; b_4 = -b_3, \; b_1 = b_2 = b_5 = b_6 = 0 \tag{9}$$

Consider truss member i for which the stress-strain relation is given by

$$\sigma_i = E_i \epsilon_i + d_i \dot{\epsilon}_i, \quad i = 1 - 6 \tag{10}$$

where σ is the normal stress, E the Young's modulus, and d the viscous damping coefficient.

It is assumed that damping is small, so that the resulting damping force in each member is much smaller than the elastic force. In other words, damping would not significantly alter the elastic strains. Thus, we use Eqs. (3) and (4) to calculate the strain rate in the presence of damping as

$$\dot{\epsilon}_i = a_i \dot{\psi},_x \tag{11}$$

for bending and

$$\dot{\epsilon}_i = b_i (\dot{w},_x + \dot{\psi}) \tag{12}$$

for the transverse shear deformation. The resulting damping forces in the truss members computed according to Eqs. (10-12) are then added to the elastic forces. As a result, Eq. (1) takes the new form

$$\begin{Bmatrix} Q \\ M \end{Bmatrix} = \begin{bmatrix} \overline{GA} & 0 \\ 0 & \overline{EI} \end{bmatrix} \begin{Bmatrix} w,_x + \psi \\ \psi,_x \end{Bmatrix} + \begin{bmatrix} C_{22} & C_{23} \\ C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \dot{w},_x + \dot{\psi} \\ \dot{\psi},_x \end{Bmatrix} \tag{13}$$

where the equivalent global damping coefficients C_{ij} are functions of the damping coefficients and dimensions of the truss members. Note that if the symmetry conditions

$$d_1 = d_2, \; d_3 = d_4, \; d_5 = d_6$$

are assumed, then $C_{23} = C_{32} = 0$.

Substituting Eqs. (8) and (9) into Eqs. (3) and (4) and then into Eq. (10), we obtain the stresses in the truss members corresponding to the bending $\psi,_x$ and the transverse shear $w,_x + \psi$, respectively. From the member stresses, the transverse shear force Q and the bending moment M are obtained as

$$Q = \frac{2L_g^2 L_c}{(L_g^2 + L_c^2)^{3/2}} A_3 E_3 (w,_x + \psi) + \frac{2L_g^2 L_c}{(L_g^2 + L_c^2)^{3/2}} A_3 d_3 (\dot{w},_x + \dot{\psi}) \tag{14}$$

$$M = \tfrac{1}{2} L_g^2 A_1 E_1 \psi,_x + \tfrac{1}{2} L_g^2 A_1 d_1 \dot{\psi},_x \tag{15}$$

Comparing Eqs. (14) and (15) with Eq. (13), we have

$$\overline{GA} = \frac{2L_g^2 L_c}{(L_g^2 + L_c^2)^{3/2}} A_3 E_3 \tag{16}$$

$$\overline{EI} = \tfrac{1}{2} L_g^2 A_1 E_1 \tag{17}$$

$$C_{22} = \frac{2L_g^2 L_c}{(L_g^2 + L_c^2)^{3/2}} A_3 d_3 \tag{18}$$

$$C_{33} = \tfrac{1}{2} L_g^2 A_1 d_1 \tag{19}$$

Following this procedure, the global stiffness and damping properties of other truss structures can be derived.

The equations of motion for the Timoshenko beam are

$$M,_x - Q = \overline{\rho I} \ddot{\psi} \tag{20}$$

$$Q,_x = \overline{m} \ddot{w} - q \tag{21}$$

where $\overline{\rho I}$ is the global rotatory inertia, $\overline{m}$ the mass per unit length of the beam, and q the transverse load per unit length. The derivation of $\overline{\rho I}$ and $\overline{m}$ was discussed in Ref. 1.

Substitution of Eq. (13) into Eqs. (20) and (21) yields the displacement-equations of motion,

$$(\overline{EI}\psi,_x),_x - \overline{GA}(w,_x + \psi) + (C_{33}\dot{\psi},_x),_x - C_{22}(\dot{w},_x + \dot{\psi}) = \overline{\rho I}\ddot{\psi} \tag{22}$$

$$[\overline{GA}(w,_x + \psi)],_x + [C_{22}(\dot{w},_x + \dot{\psi})],_x = \overline{m}\ddot{w} - q \tag{23}$$

III. Parameter Estimation: The Gradient Method

The Timoshenko beam model developed in the previous section is an idealized one. In the actual operation, due to the added controller mass, defects in fabrication of the structures, and environmental effects on material properties, the continuum model may need constant updating in its equivalent global properties.

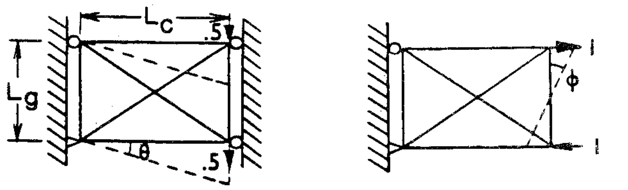
There exist many well-known methods for parameter estimation such as the gradient, Newton-Raphson, and random search methods.[4] The random search technique was successfully used in elastic truss beams.[3,4] In this paper, the gradient method is employed.[5,6]

The Timoshenko beam equations (22) and (23) and the measurement equation can be expressed in the form

$$\ddot{z} = \tilde{D}\dot{z} + \tilde{K}z + u, \quad x\epsilon\Omega, \quad t\epsilon T$$

$$y = Cz \tag{24}$$

where $z = (z_1, z_2)^T = (w, \psi)^T$ is the distributed state vector, $u = (\overline{m}^{-1}q, 0)$ is a ρ vector control input, $t\epsilon T (= (t_0, t_f))$ is the



SUBSTRUCTURE IN SHEAR SUBSTRUCTURE IN BENDING

Fig. 1 Substructure in transverse shear and bending deformations.

time, x is the spatial coordinate in domain Ω, and D and $\tilde{K}$ are the linear matrix differential operators in Ω given by

$$\tilde{D}=\begin{bmatrix} \beta_4 D^2 & \beta_4 D \\ -\beta_5 D & \beta_6 D^2-\beta_5 \end{bmatrix}, \quad D=\frac{\partial}{\partial x} \tag{25}$$

$$\tilde{K}=\begin{bmatrix} \beta_1 D^2 & \beta_1 D \\ -\beta_2 D & \beta_3 D^2-\beta_2 \end{bmatrix} \tag{26}$$

in which

$$\beta \triangleq (\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6)^T$$
$$=(\overline{GA}/\overline{m},\overline{GA}/\overline{\rho I},\overline{EI}/\overline{\rho I},\overline{C_{22}}/\overline{m},\overline{C_{22}}/\overline{\rho I},\overline{C_{33}}/\overline{\rho I})^T$$

is a vector of all the parameters in the simple continuum model, and C is a finite-dimensional influence matrix function or integral operator that is generally independent of the parameter vector β.

Of practical interest are the following boundary conditions:

1) Fixed-fixed beam

$$z(0,t)=z(\ell,t)=0 \tag{27}$$

2) Simply supported beam

$$\Gamma z(0,t)=\Gamma z(\ell,t)=0 \tag{28}$$

where

$$\Gamma=\begin{bmatrix} 1 & 0 \\ 0 & D \end{bmatrix} \tag{29}$$

3) Cantilever beam

$$z(\ell,t)=\Gamma z(0,t)=0 \tag{30}$$

where

$$\Gamma=\begin{bmatrix} 1 & -1 \\ 0 & D \end{bmatrix} \tag{31}$$

The output error is given by

$$e(t)=y(t)-\hat{y}(t)=(y_1-\hat{y}_1,\ldots,y_m-\hat{y}_m) \tag{32}$$

where $\hat{y}$ is the actual measurement vector.

The error cost function is defined by

$$J(\beta)=\frac{1}{2t_f}\int_0^{t_f} e^T R e \mathrm{d}t \tag{33}$$

where R is a positive definite weighting matrix. The objective of updating the simple model is to adjust β so that $J(\beta)$ achieves the minimum.

The variation of the cost function can be given by[5]

$$\delta J=\frac{1}{t_f}\int_0^{t_f} e^T R\left(\frac{\partial y}{\partial \beta}\right)\delta\beta \mathrm{d}t \tag{34}$$

since $\hat{y}$ does not depend on β.

If δJ could be made negative definite by properly choosing $\delta\beta$, then J would approach a minimum asymptotically. Selecting

$$\delta\beta=-\alpha\left[\int_0^{t_f}\left(\frac{\partial y}{\partial \beta}\right)^T R e \mathrm{d}t\right] \tag{35}$$

with a positive scalar α gives

$$\delta J=\frac{\alpha}{t_f}\int_0^{t_f} e^T R\left(\frac{\partial y}{\partial \beta}\right)\mathrm{d}t\int_0^{t_f}\left(\frac{\partial y}{\partial \beta}\right)^T R e \mathrm{d}t \tag{36}$$

This expression is at least negative semidefinite, but need not be negative definite, depending on the number of parameters and available measurements. Discussions relative to identifiability and observability of the solutions of the model can be found in Ref. 3.

In Eq. (34), the weighting matrix R must be properly selected, as in all gradient adjustment schemes, according to the accuracy requirement of the system parameters β. The constant α must also be properly selected. If α is too large, this scheme may diverge. On the other hand, if α is too small, then β may converge slowly.

The sensitivity matrix $(\partial y/\partial\beta)$ must be computed before this scheme can be implemented. From Eq. (24), we obtain

$$\frac{\partial y}{\partial \beta}=C\frac{\partial z}{\partial \beta} \tag{37}$$

Differentiating the model equation (24) with respect to β_i and interchanging $\partial/\partial\beta_i$ and $\mathrm{d}/\mathrm{d}t$ yields

$$\ddot{s}_i-\tilde{D}\dot{s}_i-\tilde{K}s_i=\frac{\partial u}{\partial \beta_i}+\left(\frac{\partial \tilde{D}}{\partial \beta_i}\right)\dot{z}+\left(\frac{\partial \tilde{K}}{\partial \beta_i}\right)z \tag{38}$$

where

$$s_i \triangleq \frac{\partial z}{\partial \beta_i} \qquad (i=1,2,\ldots,6) \tag{39}$$

Since the initial conditions for the model can be selected independently from $\delta\beta(t=0)$, the initial condition for the sensitivity vector s_i can be set as

$$s_i(0)=\mathrm{d}s_i(0)/\mathrm{d}t=0 \tag{40}$$

If the forcing solution u does not depend on β_i, Eq. (36) reduces to

$$\ddot{s}_i-\tilde{D}\dot{s}_i-\tilde{K}s_i=\left(\frac{\partial \tilde{D}}{\partial \beta_i}\right)\dot{z}+\left(\frac{\partial \tilde{K}}{\partial \beta_i}\right)z \tag{41}$$

Denoting

$$\eta=\begin{bmatrix} z \\ \dot{z} \end{bmatrix}, \quad \gamma_i=\begin{bmatrix} s_i \\ \dot{s}_i \end{bmatrix}, \quad A=\begin{bmatrix} 0 & I \\ \tilde{K} & \tilde{D} \end{bmatrix}$$

$$v=\begin{bmatrix} 0 \\ u \end{bmatrix}, \quad F_i=\begin{bmatrix} 0 & 0 \\ \partial\tilde{K}/\partial\beta_i & \partial\tilde{D}/\partial\beta_i \end{bmatrix} \tag{42}$$

Eqs. (24) and (41) can be converted into a set of first-order differential equations

$$\dot{\eta}=A\eta+v \tag{43}$$

$$\gamma_i=A\gamma_i+F_i\eta \tag{44}$$

It is obvious that the gradient method consists of 1) solving the sensitivity equation (44) and the system equation (43) with a specific forcing function u; 2) using Eq. (35) to evaluate the increment of parameter β; 3) computing the cost function J of Eq. (33); and 4) repeating the procedure by using updated parameters if the error cost function continues to decrease.

In general, the parameters may be of different magnitudes. A parameter normalization scheme developed in the sequel can be used to minimize the numerical error. Let $\beta(I)$ denote the parameter vector obtained from the Ith iteration. The normalized parameter vector $\tilde{\beta}(I)$ can then be written as

$$\tilde{\beta}(I)=\beta(I)/\beta(I-1) \qquad I=1,2,...$$

where $\beta(0)$ is the initial estimate from the simple model and $\tilde{\beta}(0)=1$. Instead of computing the sensitivity equation with respect to $\beta(I)$, Eq. (44) is solved with the normalized parameter vector $\tilde{\beta}(I)$, which is always close to one.

IV. Example

Consider a simply supported truss beam for which the dimension and properties of the truss members are given in Fig. 2. In this figure, A_c, A_g, and A_d indicate the member cross-sectional areas, ℓ the length of the beam, E the modulus of elasticity, ρ the mass density, and d the viscous damping coefficient.

The equivalent bending and transverse shear rigidities and the equivalent global damping coefficients for the Timoshenko beam model are computed from Eqs. (16-19) and the equivalent mass $\overline{m}$ and rotary inertia $\overline{\rho I}$ are obtained using the procedure suggested by Ref. 1. We have

$$\overline{GA}=1.468\times10^6\ \mathrm{N},\quad \overline{EI}=7.17\times10^7\ \mathrm{N\cdot m^2}$$

$$\overline{m}=0.875\ \mathrm{N\cdot s^2/m^2},\quad \overline{\rho I}=3.55\ \mathrm{N\cdot s^2}$$

$$C_{22}=8.24\times10^3\ \mathrm{N\cdot s},\quad C_{33}=4.0\times10^5\ \mathrm{N\cdot m^2\cdot s} \tag{45}$$

Substituting the mode shape of deflection, which satisfies the boundary conditions of Eq. (38) for the simply supported Timoshenko beam,

$$w=\sum_{r=1}^{N} W_r \sin\mu_r x;\quad \mu_r=\frac{r\pi}{\ell} \tag{46}$$

$$\psi=\sum_{r=1}^{N}\psi_r\cos\mu_r x \tag{47}$$

into Eqs. (43-44) with the aid of Eqs. (24) and (42) yields

$$\dot{\eta}_r=A_r\eta_r+v_r \tag{48}$$

$$\dot{\gamma}_{ir}=A_r\gamma_{ir}+F_{ir}\eta_r;\qquad r=1,...,N \tag{49}$$

where

$$\eta_r=(W_r,\Psi_r,\dot{W}_r,\dot{\Psi}_r)^T$$

$$\gamma_{ir}=\left(\frac{\partial W_r}{\partial\beta_i},\frac{\partial\Psi_r}{\partial\beta_i},\frac{\partial\dot{W}_r}{\partial\beta_i},\frac{\partial\dot{\Psi}_r}{\partial\beta_i}\right)^T \tag{50}$$

$$A_r=\begin{bmatrix}0&0&1&0\\0&0&0&1\\-\beta_1\mu_r^2&-\beta_1\mu_r&-\beta_4\mu_r^2&-\beta_4\mu_4\\-\beta_2\mu_r&-\beta_3\mu_r^2-\beta_2&-\beta_5\mu_r&-\beta_6\mu_r^2-\beta_5\end{bmatrix} \tag{51}$$

$$v_r=\left(0,0,\int_0^\ell\frac{2}{\overline{m}\ell}q(x,t)\sin\mu_r x\mathrm{d}x,0\right)^T \tag{52}$$

and

$$F_{ir}=\frac{\partial A_r}{\partial\beta_i} \tag{53}$$

Damped Free Vibration

For free vibration ($q=0$), we assume the amplitude W and Ψ_r in Eqs. (46) and (47) to be of the form $e^{\lambda t}$. The frequency equation is obtained as

$$\lambda^4+h_3\lambda^3+h_2\lambda^2+h_1\lambda+h_0=0 \tag{54}$$

where

$$h_0=\beta_1\beta_3\mu_r^4$$

$$h_1=(\beta_1\beta_6+\beta_3\beta_4)\mu_r^4$$

$$h_2=(\beta_1+\beta_3)\mu_r^2+\beta_2+\beta_4\beta_6\mu_r^4$$

$$h_3=(\beta_4+\beta_6)\mu_r^2+\beta_5$$

The biquadratic equation (54) can be solved numerically yielding an infinite number of solutions λ in the form

$$\lambda_r^{(1)}=-\delta_r^{(1)}\pm i\omega_r^{(1)},\qquad i=\sqrt{-1} \tag{55}$$

$$\lambda_r^{(2)}=-\delta_r^{(2)}\pm i\omega_r^{(2)} \tag{56}$$

where δ_r and ω_r are positive quantities. The two roots $\lambda_r^{(1)}$ and $\lambda_r^{(2)}$ represent two different branches of deformation with one frequency ω_r much larger than $\omega_r^{(1)}$. The branch with the lower frequency is of practical interest and henceforth will be used for further discussion.

Table 1 presents the natural frequencies of the first five modes (of the lower branch) for both undamped and damped systems. The natural frequencies obtained using the full-scale finite element method (modeling each truss member by a finite element) are also shown in the table. The undamped natural frequencies are within 5% of those predicted by the finite element.

Transient Vibration

Let $a_r^{(j)}\pm ib_r^{(j)}$ be the eigenvectors corresponding to the eigenvalues $-\delta_r^{(j)}\pm i\omega_r^{(j)}$ $(j=1,2)$. Since A_r [see Eq. (51)] is

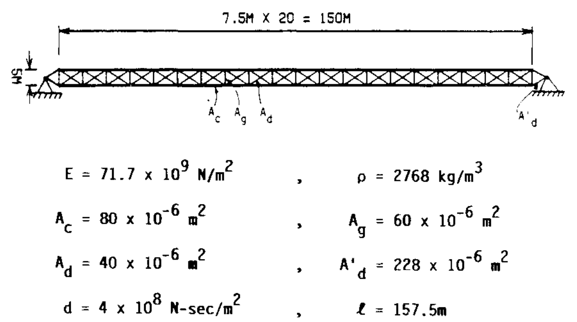


Fig. 2 Simply supported truss beam. $E=71.7\times10^9$ **N/m²,** $e=2768$ **kg/m³,** $A_c=80\times10^{-6}$ **m²,** $A_g=60\times10^{-6}$ **m²,** $A_d=40\times10^{-6}$ **m²,** $A_d^1=228\times10^{-6}$ **m²,** $d=4\times10^8$ **N·s/m²,** $\ell=157.5$ **m.**

Table 1 Natural frequencies (rad/s) and damping ratios (for $d=4\times10^8$ N·s/m²)

	Undamped frequency		Damped Timoshenko beam	
Mode	Finite element	Timoshenko beam	Frequency	Damping ratio, %
1	3.68	3.56	3.54	0.993
2	14.35	13.84	13.83	3.861
3	30.97	29.75	29.63	8.340
4	52.18	49.95	49.43	14.10
5	76.62	73.20	71.58	20.91

Table 2 Simple and updated models for the truss beam[a]

Parameters	Simple model		Updated simple model	
β_1	1.66857×10^6		1.69925×10^6	
β_2	4.11268×10^5		4.03717×10^5	
β_3	2.01972×10^7		2.05620×10^7	
β_4	9.41714×10^3		9.41619×10^3	
β_5	2.32113×10^3		2.32137×10^7	
β_6	1.12676×10^5		1.12665×10^5	
Least-squares error	6.732999×10^{-5}		1.60675×10^{-6}	
Mode \ Freq.	Damping, δ	Frequency, rad/s	Damping, δ	Frequency, rad/s
1	3.54×10^{-2}	3.564	3.54×10^{-2}	3.661
2	5.34×10^{-1}	13.83	5.33×10^{-1}	14.19
3	2.471	29.63	2.467	30.37
4	6.971	49.43	6.962	50.59
5	14.97	71.58	14.96	73.18

[a]Measurement location = 78.75 m, measurement time step = 0.05 s. Actuator location = 78.75 m, step load duration = 2.557 s. Number of modes for simple model = 5. Total simulation time 10 s.

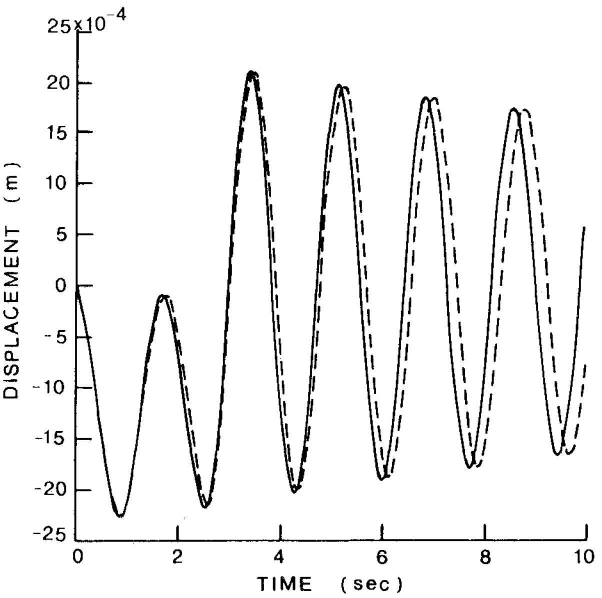


Fig. 3 Displacement response history for step load of duration $1.5T_0$ (—— finite element solution, ----- Timoshenko beam).

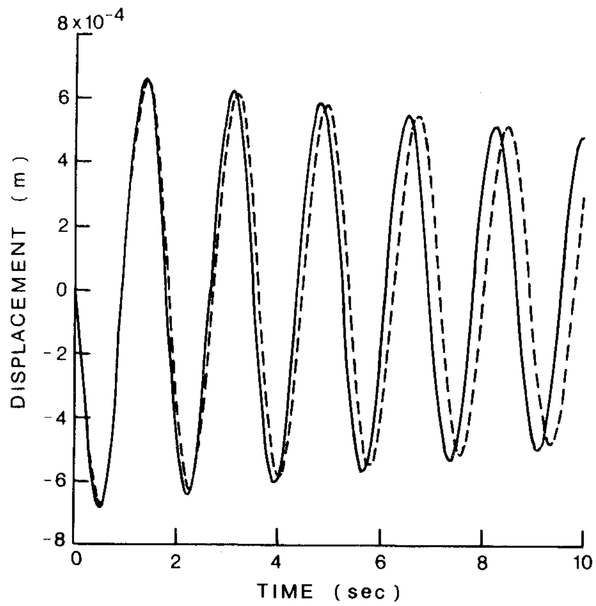


Fig. 4 Displacement response history for step load of duration $0.1T_0$ (—— finite element solution, ----- Timoshenko beam).

a real matrix, there exists a real transformation matrix T with basis $a_r^{(1)}, b_r^{(1)}, a_r^{(2)}, b_r^{(2)}$ such that $\tilde{A}_r = T^{-1}A_rT$ is a canonical matrix with the diagonal blocks $\{\tilde{A}_r^{(1)}, A_r^{(2)}\}$ in which $A_r^{(j)}$ is the real quasidiagonal matrix with nonzero diagonal block[7]

$$\begin{bmatrix} -\delta_r^{(j)} & \omega_r^{(j)} \\ -\omega_r^{(j)} & -\delta_r^{(j)} \end{bmatrix} \tag{57}$$

Defining $\tilde{\eta}_r = T\eta_r$ and $\tilde{\gamma}_{ir} = T\gamma_{ir}$, we can rewrite Eqs. (48) and (49) as

$$\dot{\tilde{\eta}}_r = \tilde{A}_r\tilde{\eta}_r + \tilde{v}_r \tag{58}$$

$$\dot{\tilde{\gamma}}_{ir} = \tilde{A}_r\tilde{\gamma}_{ir} + \tilde{F}_{ir}\tilde{\eta}_r \tag{59}$$

where

$$\tilde{v}_r = T^{-1}v_r, \qquad \tilde{F}_{ir} = T^{-1}\tilde{F}_{ir}T \tag{60}$$

Due to its simple form, the system of Eqs. (58) and (59) can be solved analytically for simple functions such as an impulsive force and a step forcing. For the purpose of illustration, we consider two cases of step loadings. The first case is a unit (1N) step force applied at the midspan of the truss beam with the duration of $1.5T_0$, where T_0 is the period (2.557 s) of the lowest undamped mode. The second case of loading is a unit step force of duration $0.1T_0$ applied at the midspan of the truss. The Laplace transform technique is used to solve Eqs. (58) and (59). The first five natural modes are included in the analysis. The truss members are assumed to have the same Young's modulus and damping coefficient.

Figures 3 and 4 show the midspan displacement histories for the two cases of loading, respectively. The solid lines indicate the solutions obtained from a full-scale finite element modeling and the dashed lines are the simple model (Timoshenko beam) solutions. It is evident that the simple model predicts the amplitude (and thus the global damping) very well. However, the phase lag becomes significant at longer time.

Model Update

We use the full-scale finite element solution as the actual response of the truss beam subjected to the prescribed loading. Assume that an actuator is placed at the midspan of

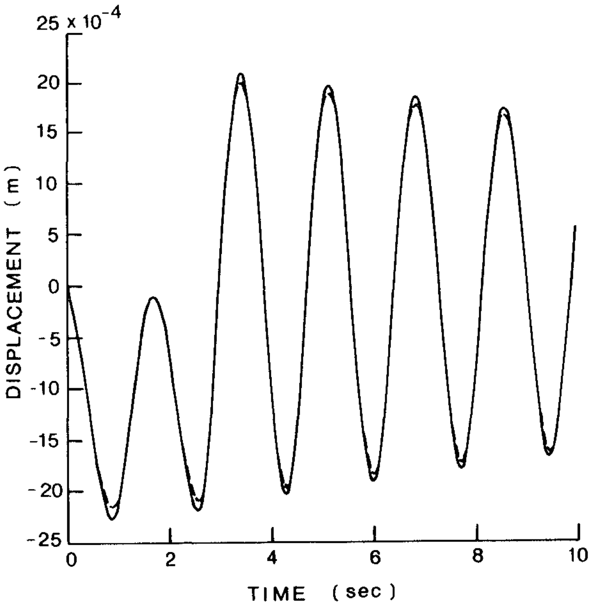


Fig. 5 Displacement response history for step load of duration $1.5T_0$ (—— finite element solution, ----- updated Timoshenko beam).

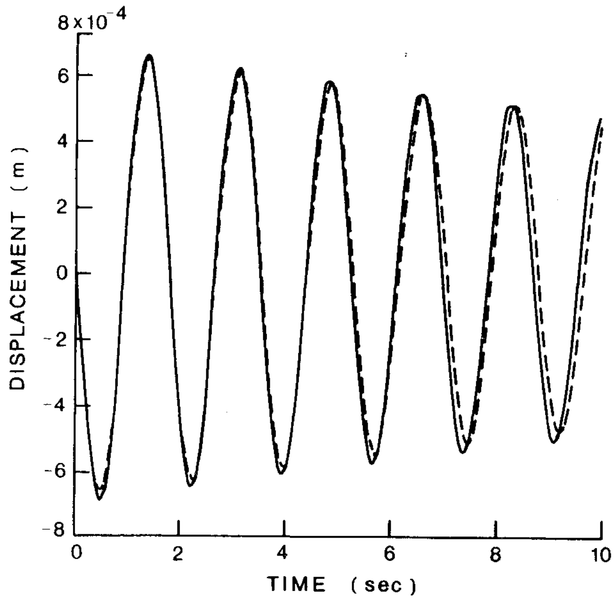


Fig. 6 Displacement response history for step load of duration $0.1T_0$ (—— finite element solution, ----- updated Timoshenko beam).

Table 3 Simple and updated models for the truss beam[a]

Parameters	Simple model	Updated model
β_1	1.66857×10^6	1.68634×10^6
β_2	4.11268×10^5	4.06927×10^5
β_3	2.01972×10^7	2.04085×10^7
β_4	9.41714×10^3	9.41748×10^3
β_5	2.32113×10^3	2.32104×10^3
β_6	1.12676×10^5	1.12680×10^5
Least-squares error	8.70424×10^{-6}	1.69280×10^{-6}

Mode \ Freq.	Damping, δ	Frequency, rad/s	Damping, δ	Frequency, rad/s
1	3.54×10^{-2}	3.564	3.54×10^{-2}	3.620
2	5.34×10^{-1}	13.83	5.34×10^{-1}	14.03
3	2.471	29.63	2.470	30.06
4	6.971	49.43	6.969	50.10
5	14.97	71.58	14.97	72.50

[a]Step load duration = 0.17046 s.

the truss to generate the prescribed force and that at the same location, a sensor picks up the displacement response history. In this case, the error e in Eq. (32) is a scalar and is calculated using the full-scale finite element solution and the simple model solution.

Using the gradient technique as discussed in Sec. III, all the parameters in β are sought to minimize the cost function J. In the search, both α and R are set equal to unity. The original values of β_i and the updated β_i are presented in Tables 2 and 3. Using the updated β_i, the natural frequencies according to the updated simple model are calculated and presented in Table 2, as is the damping factor δ_r for each mode. The same example with different forcing function has been used in Ref. 4 using the random search method for parameter estimation. Results are in good agreement with Tables 2 and 3. However, the random search method uses more computational time than the gradient technique.

Figures 5 and 6 show the displacement response histories for 10 s at the midspan for the two previously described step loadings, respectively. It is seen that the updated model has reduced the phase lag significantly, especially for the case of longer loading duration. If the least-squares error tolerance in the minimization of the cost function is set at an even smaller value, the accuracy of the updated model may further improve.

V. Conclusions

A Timoshenko beam has been used to model truss beams with damping. Procedures for deriving the global damping properties from those of the individual truss members have been presented. Free and transient vibrations were studied using the Timoshenko beam model as well as the full-scale finite element method. Comparisons of these results showed that the Timoshenko beam thus constructed is able to describe the dynamic response of damped truss structures fairly accurately.

The Timoshenko beam model having relatively few structural parameters offers an efficient way for system identifications. In this paper, a truss beam subjected to step loads was used as an example to illustrate the parameter estimation procedures using the Timoshenko beam model. It was demonstrated that the true beam properties can be easily identified using these procedures and that the updated Timoshenko beam model improves over the original one in predicting dynamic responses.

Acknowledgment

The authors are grateful to Serge Abrate of Purdue University for performing the full-scale finite element analysis. The computer program for parameter estimation was written by Catherine Herstrom of NASA Langley Research Center.

References

[1]Sun, C. T., Kim, B. J., and Bogdanoff, J. L., "On the Derivation of Equivalent Models for Beam- and Plate-Like Structures in Dynamic Analysis," *Proceedings, 1981 AIAA Dynamic Specialists Conference*, AIAA, New York, 1981, pp. 523-532.

[2]Noor, A. K., Anderson, M. S., and Green, W. H., "Continuum Models for Beam- and Plate-Like Lattice Structures," *AIAA Journal*, Vol. 16, Dec. 1978, pp. 1219-1228.

[3]Juang, J. N. and Sun, C. T., "System Identification of Large Flexible Structures by Using Simple Continuum Models," *Journal of the Astronautical Sciences*, Vol. 31, Jan.-March 1983, pp. 77-98.

[4]Sun, C. T. and Juang, J. N., "Parameter Estimation in Truss Beams Using Timoshenko Beam Model with Damping," *Proceedings of the Workshop on Applications of Distributed System Theory to the Control of Large Space Structures*, edited by G. Rodriquez, Jet Propulsion Laboratory, Pasadena, CA, Pub. 83-46, July 1983.

[5]Brogan, W. L., *Modern Control Theory*, Quantum Publishers, Inc., New York, 1974.

[6]Balakrishnan, A. V., "Modeling and Identification Theory: A Flight Control Application," *Theory and Application of Variable Structure Systems*, Academic Press, New York, 1972.

[7]Juang, J. N., Sae-Ung, S., and Yang, J. N., "Active Control of Large Building Structures," *Structural Control*, edited by H. H. E. Leipholz, North-Holland Publishing Co., Amsterdam, 1980.